

Zero-Momentum Cyclotron Spin-Flip Mode in a Spin-Unpolarized Quantum Hall System

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We report on a study of the zero-momentum cyclotron spin-flip excitation in the $\nu=2$ quantum Hall regime. Using the excitonic representation the excitation energy is calculated up to the second order Coulomb corrections. A considerable negative exchange shift relative to the cyclotron gap is established for cyclotron spin-flip excitations in the spin-unpolarized electronic system. Under these conditions this type of states presents the *lowest-energy* excitations. For a fixed filling factor ($\nu=2$) the energy shift is independent of the magnetic field which is in agreement with recent experimental observations.

PACS numbers: 73.43.Lp, 78.66.Fd

1. It is well known that in a translationally invariant two-dimensional electron system Kohn's theorem¹ prohibits coupling of a homogeneous external perturbation to collective excitations of the electrons. As a result, the energy of cyclotron excitations (CE) at zero-momentum has no contribution from Coulomb interaction and the dispersion of CE starts from the cyclotron gap. In addition to inter-Landau-levels cyclotron excitations [magnetoplasma (MP) mode] there are two other branches of collective excitations in the system of 2D-electrons: intra-Landau levels spin-flip (SF) excitations (spin-waves) and inter-Landau-levels combined cyclotron spin-flip excitations (CSFE's). In the case of SF excitations, there exists Larmor's theorem which forbids any contribution from Coulomb interaction to the excitation energy at zero-momentum in spin rotationally invariant systems (see, e.g., Ref. 2). However, in contrast to the CE and SF excitations, there are no symmetry reasons for the absence of many-body corrections to the zero-momentum energy of CSFE's. Moreover, it is well established now both theoretically and experimentally³ that for the spin-polarized electron system ($\nu=1$) the energy of cyclotron spin-flip excitations is strongly shifted to higher values relative to the cyclotron gap due to the exchange interaction. Therefore, the energy of combined cyclotron spin-flip excitations is a very convenient tool to probe many-body effects, for example, in the inelastic light scattering measurements performed at zero momentum. The sensitivity of CSFE energy at $\mathbf{q}=0$ to many-body effects strongly depends on the spin polarization of the electron system. For the spin-unpolarized electron

system ($\nu=2$), theory² developed within the first order perturbation approach in terms of the parameter $r_c = E_C/\hbar\omega_c$ (E_C is the characteristic Coulomb energy, ω_c is the cyclotron frequency) predicts a zero many-body contribution to the zero-momentum energy of CSFE. This result is in contradiction with recent experimental data.⁴ We show below that calculation of the CSFE zero-momentum energy of for the $\nu=2$ system performed to within the second order Coulomb corrections yields a considerable negative exchange shift relative to the cyclotron gap.

The studied system is characterized by exact quantum numbers S , S_z and \mathbf{q} and by a non-exact but ‘good’ quantum number δn corresponding to the change of the single-electron energy $\hbar\omega_c\delta n$ with an excitation. The relevant excitations with $\mathbf{q}=0$ and $\delta n=1$ may be presented in the form $\hat{K}_{S,S_z}^\dagger|\mathbf{0}\rangle$, where $|\mathbf{0}\rangle$ is the ground state and \hat{K}_{S,S_z}^\dagger are “raising” operators: $\hat{K}_{0,0}^\dagger = \sum_{np\sigma}\sqrt{n+1}c_{n+1,p,\sigma}^\dagger c_{n,p,\sigma}$, $\hat{K}_{1,0}^\dagger = \sum_{np\sigma}\sqrt{n+1}(-1)^\sigma c_{n+1,p,\sigma}^\dagger c_{n,p,\sigma}$ and $\hat{K}_{1,+/-}^\dagger = \sum_{np}\sqrt{n+1}c_{n+1,p,\uparrow/\downarrow}^\dagger c_{n,p,\downarrow/\uparrow}$, [$c_{n,p,\sigma}$ is the Fermi annihilation operator corresponding to the Landau-gauge state (n,p) and spin index $\sigma=\uparrow,\downarrow$]. The commutators with the kinetic-energy operator \hat{H}_1 are $[\hat{H}_1, \hat{K}_{S,S_z}^\dagger] \equiv \hbar\omega_c \hat{K}_{S,S_z}^\dagger$. (The total Hamiltonian is $\hat{H}_{\text{tot}} = \hat{H}_1 + \hat{H}_{\text{int}}$, where \hat{H}_{int} is the exact Coulomb-interaction Hamiltonian.) If $|\mathbf{0}\rangle$ is unpolarized, we have $\hat{S}^2 \hat{K}_{S,S_z}^\dagger|\mathbf{0}\rangle \equiv S(S+1)\hat{K}_{S,S_z}^\dagger|\mathbf{0}\rangle$, $\hat{S}_z \hat{K}_{S,S_z}^\dagger|\mathbf{0}\rangle \equiv S_z \hat{K}_{S,S_z}^\dagger|\mathbf{0}\rangle$ and besides get the identity $\langle 0|\hat{K}_{S,S_z}[\hat{H}_{\text{int}}, \hat{K}_{S,S_z}^\dagger]|\mathbf{0}\rangle \equiv 0$ ($|\mathbf{0}\rangle$, to describe the zero’t h order ground state). The latter determines the first-order Coulomb corrections vanishing both for the $S=0$ MP mode and for the $S=1$ triplet states corresponding to the combined spin-cyclotron excitation. At the same time $[\hat{H}_{\text{int}}, \hat{K}_{0,0}^\dagger] \equiv 0$ ¹ but $[\hat{H}_{\text{int}}, \hat{K}_{1,S_z}^\dagger] \neq 0$ which means that the MP mode indeed has no exchange energy calculated to *any order* in r_c , whereas the triplet states have the exchange correction even in terms of r_c^2 .

The second-order correction, $\Delta E_{\text{SF}} \sim \hbar\omega_c r_c^2$, does not depend on the magnetic field since $E_C = \alpha e^2/\varepsilon l_B$. The renormalization factor, α , is determined by the size-quantized wave function of electrons confined to the quantum well (QW). In the ideal 2D case $\alpha = 1$. However, in experiments with comparatively wide QW’s we expect a well reduced value of α . Our *analytical* calculation of the second order correction to the CSFE energy is performed in terms of r_c assumed to be small.

All three triplet states have certainly the same exchange energy, and it is sufficient to calculate this, e.g., for the CSFE with $S=1$ and $S_z=-1$. The obtained result confirms experimental observations.

2. The most adequate approach to the integer-quantum-Hall calculations is based on the Excitonic Representation (ER)^{5,6} technique (see also, e.g., Refs. 7). The latter means that instead of single-electron states belonging to a continuously degenerate Landau level (LL)

we employ the exciton states $\mathcal{Q}_{ab\mathbf{q}}^\dagger|0\rangle$ as the basis set. Here $|0\rangle$ is the ground state found in the zero approximation in r_c (it remains also the same even calculated within the framework of the mean field approach). The exciton creation operator is defined as^{5,6,7,8}

$$\mathcal{Q}_{ab\mathbf{q}}^\dagger = \frac{1}{\sqrt{N_\phi}} \sum_p e^{-iq_x p} b_{p+\frac{q_y}{2}}^\dagger a_{p-\frac{q_y}{2}}. \quad (1)$$

$N_\phi = A/2\pi l_B^2$ stands for the number of magnetic flux quanta, $\mathbf{q} = (q_x, q_y)$ is the 2D wave vector in units of $1/l_B$. Binary indexes a and b present both the LL number and spin index. [I.e. $a = (n_a, \sigma_a)$, and a_p in Eq. (1) stands for the corresponding annihilation operator; when exploiting below the notation $a = n$ or $a = \bar{n}$ as sublevel indexes, this means that $a = (n, \uparrow)$ or $a = (n, \downarrow)$, respectively.] The annihilation exciton operator is $\mathcal{Q}_{ab\mathbf{q}} \equiv \mathcal{Q}_{ba-\mathbf{q}}^\dagger$. The commutation rules define a special Lie algebra:^{5,7,8}

$$\begin{aligned} [\mathcal{Q}_{cd\mathbf{q}_1}^\dagger, \mathcal{Q}_{ab\mathbf{q}_2}^\dagger] \equiv & N_\phi^{-1/2} \left[e^{-i(\mathbf{q}_1 \times \mathbf{q}_2)z/2} \delta_{b,c} \mathcal{Q}_{ad\mathbf{q}_1+\mathbf{q}_2}^\dagger \right. \\ & \left. - e^{i(\mathbf{q}_1 \times \mathbf{q}_2)z/2} \delta_{a,d} \mathcal{Q}_{cb\mathbf{q}_1+\mathbf{q}_2}^\dagger \right], \end{aligned} \quad (2)$$

where $\delta_{a,b} = \delta_{n_a, n_b} \delta_{\sigma_a, \sigma_b}$ is the Kronecker symbol. In the $\nu = 2$ case we get the following identity: $N_\phi^{-1/2} \mathcal{Q}_{aa\mathbf{q}}^\dagger|0\rangle \equiv \delta_{\mathbf{q},0} (\delta_{a,0} + \delta_{a,\bar{0}}) |0\rangle$.

The advantage of the exciton states lies in the fact that an essential part of the Coulomb interaction Hamiltonian may be diagonalized in this basis. In the perturbative approach the excitonically diagonalized part \hat{H}_{ED} should be included into the unperturbed Hamiltonian $\hat{H}_0 = \hat{H}_1 + \hat{H}_{\text{ED}}$ and only the off-diagonal part $\hat{\mathcal{H}}_{\text{int}} = \hat{H}_{\text{int}} - \hat{H}_{\text{ED}}$ is considered as a perturbation.⁵ In the excitonic basis the LL degeneracy becomes well lifted because now there are Coulomb corrections (depending on the \mathbf{q} modulus) to the energies of the basis states. It is useful to take into account that all terms of the relevant $\hat{\mathcal{H}}_{\text{int}}$ part may be presented in the form (cf. Ref. 5)

$$\begin{aligned} \hat{\mathcal{H}}_{\text{int}} = \frac{e^2}{2\epsilon l_B} \sum_{\mathbf{q}, a, b, c, d} V(\mathbf{q}) & \left[h_{n_a n_b}(\mathbf{q}) \delta_{\sigma_a, \sigma_b} \mathcal{Q}_{ab\mathbf{q}}^\dagger \right] \\ & \times \left[h_{n_c n_d}(-\mathbf{q}) \delta_{\sigma_c, \sigma_d} \mathcal{Q}_{cd-\mathbf{q}}^\dagger \right]. \end{aligned} \quad (3)$$

Here $2\pi V(\mathbf{q})$ is the dimensionless 2D Fourier component of the averaged Coulomb potential (in the ideal 2D case $V = 1/q$), and $h_{kn}(\mathbf{q}) = (k!/n!)^{1/2} e^{-q^2/4} (q_-)^{n-k} L_k^{n-k}(q^2/2)$ are the ER “building-block” functions (L_k^n is the Laguerre polynomial, $q_\pm = \mp \frac{i}{\sqrt{2}}(q_x \pm iq_y)$; cf. also Ref. 2 and Refs. 5,7). The functions h_{kn} satisfy the identity: $h_{kn}^*(\mathbf{q}) \equiv h_{nk}(-\mathbf{q})$.

At the $\nu = 2$ filling the CSFE state calculated within the zero order in $\hat{\mathcal{H}}_{\text{int}}$ is simply $|SF\rangle = \hat{Q}_{0\bar{1}}^\dagger|0\rangle$ (the notation $\hat{Q}_{ab\mathbf{0}}^\dagger = \hat{Q}_{ab}^\dagger$ is employed). We thus have $\hat{H}_{\text{ED}} \mathcal{Q}_{0\bar{1}}^\dagger|0\rangle = \langle 0|\mathcal{Q}_{ab\mathbf{q}} \hat{\mathcal{H}}_{\text{int}} \mathcal{Q}_{0\bar{1}}^\dagger|0\rangle = 0$ for any indexes \mathbf{q} and ab (one could check it directly using the ER approach; see also Ref. 2). Action of $\hat{\mathcal{H}}_{\text{int}}$ on the $|SF\rangle$ state leads to two- and or even three-exciton states. Therefore the excitonic basis should be extended.

3. In principle, there are eight different kinds of possible two-exciton states at $\nu = 2$. In our case the relevant ones are those corresponding to spin numbers $S_z = -1$ and $S = 0$, namely: $|\nu, 1\rangle = \mathcal{Q}_{0n_2 - \mathbf{q}_\nu}^\dagger \mathcal{Q}_{0\bar{n}_1 \mathbf{q}_\nu}^\dagger |0\rangle$, $|\nu, 2\rangle = \mathcal{Q}_{0\bar{n}_2 - \mathbf{q}_\nu}^\dagger \mathcal{Q}_{0\bar{n}_1 \mathbf{q}_\nu}^\dagger |0\rangle$, $|\nu, 3\rangle = \frac{1}{2} \mathcal{Q}_{0n_2 - \mathbf{q}_\nu}^\dagger \mathcal{Q}_{0n_1 \mathbf{q}_\nu}^\dagger |0\rangle$, $|\nu, 4\rangle = \mathcal{Q}_{0n_2 - \mathbf{q}_\nu}^\dagger \mathcal{Q}_{0\bar{n}_1 \mathbf{q}_\nu}^\dagger |0\rangle$, and $|\nu, 5\rangle = \frac{1}{2} \mathcal{Q}_{0\bar{n}_2 - \mathbf{q}_\nu}^\dagger \mathcal{Q}_{0\bar{n}_1 \mathbf{q}_\nu}^\dagger |0\rangle$ (certainly only the states with zero total momentum should be considered). We have used here ν as a composite index corresponding to the set $(n_1, n_2, \mathbf{q}_\nu)$. The two-exciton states of different types are orthogonal, i.e. $\langle I, \nu | \mu, J \rangle = 0$ if $I \neq J$ [μ is the set $(m_1, m_2, \mathbf{q}_\mu)$, below $\lambda = (l_1, l_2, \mathbf{q}_\lambda), \dots$]. However, within the same type their orthogonalization rules should be defined in a special way.

First, let us consider a combination

$$\sum_{\nu} f_{\nu} |\nu, I\rangle \quad (4)$$

(summation is performed over all components of the composite index). In this case the function $f_{\nu} = f(n_1, n_2, \mathbf{q}_\nu)$ formally turns out to be non-uniquely defined because only a certain transform of this has a physical meaning. Indeed, actually only a projection of the sum (4) onto a certain two-exciton state $|\mu, J\rangle$ would be of any sense. With the help of commutation rules we obtain

$$\sum_{\nu} f_{\nu} \langle J, \mu | \nu, I \rangle \equiv \delta_{I,J} \{f_{\mu}\}_I, \quad (5)$$

(cf. Ref. 5). Here the curly brackets mean the transform

$$\begin{aligned} \{f_{\nu}\}_I &= f_{\nu} - N_{\phi}^{-1} \sum_{\lambda} \mathcal{F}_{\nu\lambda}^{(I)} f_{\lambda}, \quad \text{if } I = 1, 2 \text{ or } 4; \\ \text{and } \{f_{\nu}\}_I &= \frac{1}{2} \left(f_{\nu} - N_{\phi}^{-1} \sum_{\lambda} \mathcal{F}_{\nu\lambda}^{(I)} f_{\lambda} \right), \quad \text{if } I = 3 \text{ or } 5. \end{aligned} \quad (6)$$

The definition of the kernels $\mathcal{F}_{\nu\lambda}^{(I)} f_{\lambda}$ is also parametrized by the kind I of the state, namely:

$$\begin{aligned} \mathcal{F}_{\nu\lambda}^{(1)} &= \mathcal{F}_{\nu\lambda}^{(3)} = \mathcal{F}_{\nu\lambda}^{(5)} \equiv \delta_{n_1, l_1} \delta_{n_2, l_2} e^{i(\mathbf{q}_\nu \times \mathbf{q}_\lambda)z}, \\ \mathcal{F}_{\nu\lambda}^{(2)} &\equiv \delta_{n_1, l_2} \delta_{n_2, l_1} e^{-i(\mathbf{q}_\nu \times \mathbf{q}_\lambda)z}, \quad \text{and } \mathcal{F}_{\nu\lambda}^{(4)} \equiv 0. \end{aligned} \quad (7)$$

Note that the transform $\{\dots\}_I$ is to within a factor equivalent to its double application: $\{\{f\}_I\}_I = K_I \{f\}_I$, where $K_1 = K_2 = 2$ and $K_3 = K_4 = K_5 = 1$. Therefore, if we replace, e.g., $f_{\nu} \rightarrow f_{\nu} + K_I \varphi_{\nu} - \{\varphi_{\lambda}\}_I$ (φ_{ν} is an arbitrary function), then this operation does not affect the combinations (4) and (5). So, only the ‘‘antisymmetrized’’ part $\{f_{\nu}\}_I$ contributes to the matrix-element calculations. The origin of this feature of the two-exciton states is related to the permutation antisymmetry of the total wave function describing the electron system studied (cf., e.g., Refs. 5,9). There is also a useful identity

$$\sum_{\nu} w(n_1, n_2) f_{\nu}^* \{g_{\nu}\}_I \equiv \sum_{\nu} w(n_1, n_2) \{f_{\nu}\}_I^* g_{\nu}, \quad (8)$$

which is valid for any kinds of the transforms $\{\dots\}_I$ if the function w in Eq. (5) is assumed to be such that $w(n_1, n_2) \equiv w(n_2, n_1)$. In particular, Eq. (5) gives the equations:

$$\langle I, \nu | \mu, J \rangle = \delta_{I,J} \{\delta^{(I)}\}_I \quad (9)$$

where

$$\begin{aligned}\delta_{\nu\mu}^{(1)} &= \delta_{\nu\mu}^{(2)} = \delta_{\nu\mu}^{(4)} \equiv \delta_{n_1, m_1} \delta_{n_2, m_2} \delta_{\mathbf{q}_\nu, \mathbf{q}_\mu} \quad \text{and} \\ \delta_{\nu\mu}^{(3)} &= \delta_{\nu\mu}^{(5)} \equiv \frac{1}{2} \left(\delta_{n_1, m_1} \delta_{n_2, m_2} \delta_{\mathbf{q}_\nu, \mathbf{q}_\mu} + \delta_{n_1, m_2} \delta_{n_2, m_1} \delta_{\mathbf{q}_\nu, -\mathbf{q}_\mu} \right).\end{aligned}$$

Summation in the $\{\delta_{\nu\mu}^{(I)}\}_I$ transform is performed over the first index: e.g. $\{\delta_{\nu\mu}^{(1)}\}_1 \equiv \delta_{\nu\mu}^{(1)} - \mathcal{F}_{\nu\mu}^{(1)}/N_\phi$, and so on.

4. The first-order corrections (in terms of $\hat{\mathcal{H}}_{\text{int}}$) to the CSFE energy are presented as an expansion over the two-exciton states $|\nu, I\rangle$ and three-exciton states $Q_{0\bar{1}}^\dagger|\nu, I\rangle$, namely:

$$\begin{aligned}|SF, \rangle &= Q_{0\bar{1}}^\dagger|0\rangle + \sum_{I=1,2} \sum_\nu C_\nu^{(I)} |\nu, I\rangle \\ &\quad + \sum_{I=3,4,5} \sum_\nu C_\nu^{(I)} Q_{0\bar{1}}^\dagger |\nu, I\rangle.\end{aligned}\tag{10}$$

A regular application of the perturbative approach¹⁰ leads to the following expression for the exchange correction to the energy: $\Delta E_{\text{SF}} = \langle SF | \hat{\mathcal{H}}_{\text{int}} Q_{0\bar{1}}^\dagger | 0 \rangle$. Substituting $|SF, \rangle$ from Eq. (10) we see that the *contribution of the two-exciton states* to the energy arises only due to the terms of Eq. (3) which do not commute with $Q_{0\bar{1}}^\dagger$:

$$\Delta E_{1-2} = \sum_{I=1,2} \sum_\nu C_\nu^{(I)*} \langle I, \nu | [\hat{\mathcal{H}}_{\text{int}}, Q_{0\bar{1}}^\dagger] | 0 \rangle.\tag{11}$$

The coefficients $C_\nu^{(I)}$ are determined by the equations

$$\sum_\mu C_\mu^{(I)} \langle I, \nu | \mu, I \rangle = - \langle I, \nu | [\hat{\mathcal{H}}_{\text{int}}, Q_{0\bar{1}}^\dagger] | 0 \rangle / \Delta_\nu\tag{12}$$

($I=1, 2$), where $\Delta_\nu = \hbar\omega_c(n_1+n_2-1) > 0$ stands for the difference of the cyclotron energies in the states $|\nu, I\rangle$ and $Q_{0\bar{1}}^\dagger|0\rangle$. Calculating the commutator in Eqs. (11)-(12) [employing the rules (2)], and then using the properties (5) and (8) of the summation over index, we obtain

$$\Delta E_{1-2} = -N_\phi^{-1} \sum_{I=1,2} \sum_\nu \{F_\nu\}_I F_\nu^* / (n_1 + n_2 - 1)\tag{13}$$

[in units of $2\text{Ry} = (e^2/\varepsilon l_B)^2/\hbar\omega_c = m_e^* e^4/\varepsilon^2 \hbar^2$], where

$$F_\nu = V(\mathbf{q}_\nu) [h_{1n_1}(\mathbf{q}_\nu) - \delta_{1,n_1} h_{00}(\mathbf{q}_\nu)] h_{0n_2}(-\mathbf{q}_\nu).\tag{14}$$

Now we calculate the contribution ΔE_{3-5} which is determined by the *three-exciton states* [see Eq. (10)]. This correction arises from the commuting part (with $Q_{0\bar{1}}^\dagger$) of $\hat{\mathcal{H}}_{\text{int}}$ acting on the state $Q_{0\bar{1}}^\dagger|0\rangle$, i.e.

$$\Delta E_{3-5} = \sum_{I=3,4,5} \sum_\nu C_\nu^{(I)*} \langle I, \nu | Q_{0\bar{1}} Q_{0\bar{1}}^\dagger \hat{\mathcal{H}}_{\text{int}} | 0 \rangle.\tag{15}$$

The equations for the coefficients are

$$\begin{aligned}\sum_\mu C_\mu^{(I)} \langle I, \nu | Q_{0\bar{1}} Q_{0\bar{1}}^\dagger | \mu, I \rangle \\ = - \langle I, \nu | Q_{0\bar{1}} Q_{0\bar{1}}^\dagger \hat{\mathcal{H}}_{\text{int}} | 0 \rangle / \tilde{\Delta}_\nu\end{aligned}\tag{16}$$

($I=3,4,5$), where $\tilde{\Delta}_\nu = \hbar\omega_c(n_1+n_2) \geq 2$. Substituting $Q_{0\bar{I}}Q_{0\bar{I}}^\dagger \equiv N_\phi^{-1/2}(Q_{00}-Q_{\bar{I}\bar{I}})+Q_{0\bar{I}}^\dagger Q_{0\bar{I}}$ into Eqs. (15)-(16) we deduce that the operator $Q_{0\bar{I}}^\dagger Q_{0\bar{I}}$ gives no contribution, whereas action of the remaining terms reduces the convolutions in Eqs. (15)-(16) to the “bra-ket” products of two-exciton states. In so doing we find a huge contribution (eventually $\sim N_\phi$) into Eq. (15) due to the commuting part of $N_\phi^{-1/2}(Q_{00}-Q_{\bar{I}\bar{I}})$, which is actually nothing else but the second order correction (in terms of r_c) to the ground state, namely: $\Delta E_0 = \sum_{\nu; I=3,4,5} C_\nu^{(I)} \langle 0 | \hat{\mathcal{H}}_{\text{int}} | \nu, I \rangle$. According to Eq. (16)

$$\{C_\nu^{(I)}\}_I = -[e^2/(\epsilon l_B \hbar \omega_c)] \{G_\nu\}_I / (n_1 + n_2) \quad (17)$$

($I=3,4,5$) with

$$G_\nu = V(\mathbf{q}_\nu) h_{0n_1}(\mathbf{q}_\nu) h_{0n_2}(-\mathbf{q}_\nu). \quad (18)$$

The non-commuting part determines the corrections to the bra-vectors in Eq. (15). For the $I=3$ states we get

$$N_\phi^{-1/2} \left[Q_{00} - Q_{\bar{I}\bar{I}}, \frac{1}{2} Q_{0n_2-\mathbf{q}_\nu}^\dagger Q_{0n_1\mathbf{q}_\nu}^\dagger \right] |0\rangle = -2|\nu, 3\rangle / N_\phi,$$

and correspondingly $-(1+\delta_{n_1,1})|\nu, 4\rangle/N_\phi$ and $-(\delta_{n_1,1}+\delta_{n_2,1})|\nu, 5\rangle/N_\phi$ at $I=4$ and $I=5$ [the identities (2) have been used]. The similar corrections to the bra-vectors in Eq. (16) do not affect the equation (17) for $C_\nu^{(I)}$.

The desirable exchange shift should be measured from corrected energy of the ground-state. We keep thus in Eq. (15) only the contribution of the non-commuting part (i.e. considering $\Delta E_{3-5} \rightarrow \Delta E_{3-5} - \Delta E_0$). Then by substituting Eq. (3) for $\hat{\mathcal{H}}_{\text{int}}$ into Eq. (15) and using again the summation rules (5) and (8) we find from Eqs. (15) and (17) the $I=3-5$ correction

$$\Delta E_{3-5} = \frac{1}{N_\phi} \sum_\nu [(2+\delta_{n_1,1}+\delta_{n_2,1}) \{G_\nu\}_3 + (1+\delta_{n_1,1}) G_\nu] G_\nu^* / (n_1 + n_2) \quad (19)$$

(in units of $2Ry$). The combination with Eq. (13) yields

$$\Delta E_{\text{SF}} = \Delta E_{1-2} + \Delta E_{3-5}. \quad (20)$$

The sum over ν in Eqs. (13) end (19) means summation over n_1 and n_2 and the integration over \mathbf{q}_ν . This is a routine procedure and the suitable sequence of operations is as follows. First we perform the summation over all of $n_1 \geq 1$ and $n_2 \geq 1$ keeping the sum $n_\nu = n_1 + n_2$ fixed. Then we make the integration over \mathbf{q}_ν . According to the above definition, the transforms $\{F_\nu\}_I$ and $\{G_\nu\}_I$ already contain an integration, therefore some terms in Eq. (8) present twofold integration over 2D vectors \mathbf{q}_λ and \mathbf{q}_ν . Really the latter, with the help of formula $(2\pi)^{-2} \iint d\mathbf{q}_1 d\mathbf{q}_2 U(q_1, q_2) (q_{1+} q_{2-})^m e^{\pm i(\mathbf{q}_1 \times \mathbf{q}_2)_z} \equiv \int_0^\infty \int_0^\infty dq_1 dq_2 (q_1 q_2)^m U(q_1, q_2) J_{\pm m}(q_1 q_2)$ (J_m is the Bessel function, U is an arbitrary function),

is reduced to integration over absolute values q_λ and q_ν . Finally the numerical summation over n_ν is performed.

In so doing, a simplifying circumstance was found: all of the twofold-integration terms cancel each other in the final combination (20). (This feature is not a general one but only inherent in our specific case.¹¹) All the rest terms result in the following expression:

$$\Delta E_{\text{SF}} = - \sum_{n=2}^{\infty} S_n \frac{1-2^{1-n}}{n(n^2-1)}, \quad \text{where} \quad (21)$$

$$S_n = \frac{2}{n!} \int_0^\infty dq q^{2n+3} V^2(q) e^{-q^2}.$$

For the ideally 2D system we have $S_n \equiv 1$, and the summation may be easily performed, yielding $\Delta E_{\text{SF}} = (\ln 2 - 1)/2 = -0.1534\dots$ (in units of 2Ry).

5. So, the shift is negative and the exchange interaction lowers thereby the CSFE energy relative to the singlet MP mode. The sign of the shift presents an expectable result. Indeed, the second-order correction to the energy of a low-lying excitation should be *presumably* negative due to the same reasons which determine the *inevitably* negative sign of the correction to the ground state energy. Another remarkable feature of the found shift is its independence of the magnetic field.

Due to the $\mathbf{q}=0$ condition the studied state is optically active and should be observed in photo-luminescent and inelastic light scattering (ILS) measurements. In the recent work 4 the ILS was studied in a single 30 nm AlGaAs/GaAs QW in the situations where $\nu=2; 4$. The triplet and MP cyclotron excitations are manifested as peaks in the ILS spectra. The measurements were performed in magnetic fields varied in a wide range, but with the filling factor kept constant. The central triplet line is shifted downward from the cyclotron energy by 0.35 meV independently of the B magnitude. Thus, a qualitative agreement with our calculation is obvious.

Quantitative comparison should be done with taking into account of finite thickness of a two-dimensional electron gas. The calculation in Fig. 1 incorporates the effect of the finite width of the 2D layer. This is carried out by writing the Coulomb vertex as $V(q) = F(qw)/q$, where the form factor $F(qw)$ is parametrized by an effective thickness w . The latter characterizes the spread of the electron wavefunction in the perpendicular direction. If the variational envelope function is chosen in the form $|\psi(z)|^2 \sim \exp(-z^2/2w^2)$, then $F(qw) = e^{w^2 q^2} \text{erfc}(wq)$ (see Ref. 12). Exactly this form factor is employed in the calculation based on Eq. (21). Taking into account the value of $\text{Ry} = 5.67 \text{ meV}$ in GaAs, we find from Fig. 1 that the agreement with the experiment is obtained at $w \approx 0.5l_B$. This is quite reasonable value for the 30 nm GaAs quantum structure.

As a concluding remark we notice that the triplet cyclotron excitation in spin-unpolarized electron system seems to have been observed earlier,¹³ although in this paper experimental

observations were related to the roton minimum and a different experimental dependence of energy shift on magnetic field was detected.

We acknowledge support by the Russian Fund of Basic Research. S. D. thanks for hospitality the Max Planck Institute for Physics of Complex Systems (Dresden), where an essential part of this work has been done.

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- ¹ W. Kohn, Phys. Rev., **123**, 1242 (1961).
 - ² C. Kallin and B. I. Halperin, Phys. Rev. B **30**, 5655 (1984).
 - ³ A. Pinczuk *et al.*, Phys. Rev. Lett. **68**, 3623 (1992).
 - ⁴ L.V. Kulik, I.V. Kukushkin, S. Dickmann, V.E. Kirpichev, A.B. Van'kov, A.L. Parakhonsky, J.H. Smet, K. v. Klitzing, and W. Wegscheider, to appear (cond-mat/0501466).
 - ⁵ S. Dickmann, Phys. Rev. B **65**, 195310 (2002).
 - ⁶ A.B. Dzyubenko and Yu.E. Lozovik, Sov. Phys. Solid State **25**, 874 (1983) [*ibid.* **26**, 938 (1984)].
 - ⁷ S. Dikman, and S. V. Iordanskii, JETP **83**, 128 (1996); S. Dickmann and Y. Levinson, Phys. Rev. B **60** 7760 (1999); S. Dickmann, Phys. Rev. B **61**, 5461 (2000); and Phys. Rev. Lett. **93**, 206804 (2004).
 - ⁸ Yu.A. Bychkov, and S.V. Iordanskii, Sov. Phys. Solid State **29**, 1405 (1987).
 - ⁹ Yu.A. Bychkov, and E.I. Rashba, JETP **58**, 1062 (1983).
 - ¹⁰ L.D. Landau and E.M. Lifschitz, *Quantum Mechanics* (Butterworth-Heinemann, Oxford, 1991).
 - ¹¹ E.g. these terms are available in the second order corections to the skyrmion-antiskyrmion gap [6] and to the energy of the hole at the $\nu=1$ filling. We remark that the hole energy correction was calculated in Ref. [6] without taking into account the twofold-integration contribution. The latter constitutes -0.00349 (in the strict 2D limit) and has to be added to the result $-\pi^2/24+\frac{3}{4}(\ln 2)^2$ presented there. The total correction is thereby -0.0544 in units of $2Ry$ (cf. the numerical result in Ref. [14]).
 - ¹² N.R. Cooper, Phys. Rev. B **55**, 1934 (1997).
 - ¹³ M.A. Eriksson *et al.*, Phys. Rev. Lett. **82**, 2163 (1999).
 - ¹⁴ S.L. Sondhi *et al.*, Phys. Rev. B **47**, 16419 (1993).

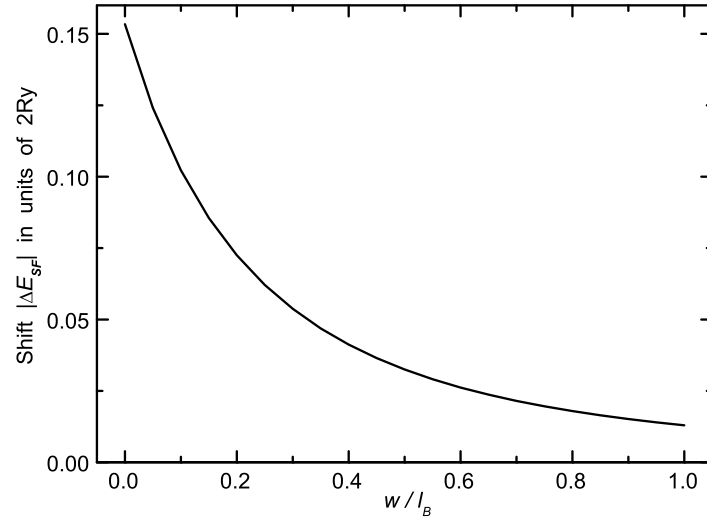


FIG. 1: The CSFE exchange shift is calculated from the formula of Eq. (21) with the modified Coulomb interaction $V(q) = q^{-1} e^{q^2 w^2} \text{erfc}(qw)$; the shift value absolute at $w=0$ is $(1 - \ln 2)/2$.